

Question 1:**[Maximum mark: 7]**

An arithmetic sequence is given by **8, 12, 16, 20, ...**

(a) Write down the value of the common difference, d . **[1]**

(b) Find

(i) u_{15} ; **[2]**

(ii) S_{15} . **[2]**

(c) Given that $u_n = 158$, find the value of n . **[2]**

Question 2**[Maximum mark: 6]**

Let $a = \log_4 b$, where $b > 0$. Write down each of the following expressions in terms of a .

(a) $\log_4(b^5)$ **[2]**

(b) $\log_4(16b)$ **[2]**

(c) $\log_{64} b$ **[2]**

Question 3**[Maximum mark: 6]**

Consider the expansion of $(3x + 2)^8$.

(a) Write down the number of terms in this expansion. **[1]**

(b) Find the coefficient of the term in x^3 . **[5]**

Question 4**[Maximum mark: 6]**

In an arithmetic sequence,

$$u_5 = 18, u_{14} = -9.$$

(a) Find the common difference. **[2]**

(b) Find the first term. **[2]**

(c) Find the sum of the first 14 terms of the sequence. **[2]**

Question 5

[Maximum mark: 5]

The fourth term, in descending powers of x , in the expansion of $(2x + p)^7$ is $560x^4$.

Find the possible values of p .

Question 6

[Maximum mark: 6]

(a) Show that

$$(3n - 1)^3 + (3n + 1)^3 = 54n^3 + 18n$$

for $n \in \mathbb{Z}$. **[3]**

(b) Hence, or otherwise, prove that the sum of the cubes of any two consecutive multiples of 3 is divisible by 18. **[3]**

Question 7

[Maximum mark: 6]

(a) Write down the value of

(i) $\log_4 64$; **[1]**

(ii) $\log_7 \left(\frac{1}{49}\right)$; **[1]**

(iii) $\log_{25} 5$. **[1]**

(b) Hence solve

$$\log_4 64 + \log_7 \left(\frac{1}{49}\right) + \log_{25} 5 = \log_{10} x.$$

[3]

Question 8

[Maximum mark: 7]

The first three terms of a geometric sequence are

$$u_1 = 5, u_2 = 15, u_3 = 45.$$

(a) Find the value of the common ratio, r . [2]

(b) Find the value of S_9 . [2]

(c) Find the least value of n such that

$$S_n > 150\,000.$$

[3]

Question 9

[Maximum mark: 6]

On 1st July 2024, Ethan invests **\$12,000** in a savings account that pays a **nominal annual interest rate of 4.8%**, compounded **monthly**.

The amount of money in Ethan's account at the end of each year follows a geometric sequence with common ratio, r .

(a) Find the value of r , giving your answer to **four significant figures**. [3]

Ethan makes no further deposits or withdrawals from the account.

(b) Find the first year in which the amount in Ethan's account will exceed **twice** the amount he invested. [3]

Question 10

[Maximum mark: 5]

Emma cycles to school each morning.

During the **first minute**, she cycles **180 metres**. In each subsequent minute, she cycles **90%** of the distance travelled during the previous minute.

The distance from Emma's home to school is **1200 metres**. Emma leaves home at **7:45 am** and must arrive at school by **7:55 am**.

Determine whether Emma will arrive at school on time. **Justify your answer using calculations.** [5]